PAUTA CONTROL 4 - MA11A-ALGEBRA

(1999)

Pregunta 1.

$$\left[\begin{array}{cccc|c|cccc} -\beta & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ -\beta & 3 & -2 & 1 & 2 & 0 & 1 & 0 & 0 \\ \beta & -2 & -1 & 1 & -1 & 0 & 0 & 1 & 0 \\ -\beta & 4 & -4 & \alpha+1 & (\alpha+\beta+3) & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c|cccc} -\beta & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & -4 & \alpha & \alpha+\beta+2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c|cccc} -\beta & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & \alpha & \alpha+\beta & 1 & -2 & 0 & 1 \end{array}\right]$$

<u>caso $\beta = 0$:</u>

$$\left[\begin{array}{cccc|c|cccc} 0 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & \alpha & \alpha & 1 & -2 & 0 & 1 \end{array}\right]$$

<u>un escalonamiento $+$:</u>

$$\left[\begin{array}{cccc|c|cccc} 0 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & -1 & 3 & -2 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & \alpha & \alpha & 1 & -2 & 0 & 1 \end{array}\right] \rightarrow$$

$$\left[\begin{array}{cccc|c|cccc} 0 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & -1 & 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 9 & -1 & 7 & -2 & 4 & 0 \\ 0 & 0 & 0 & \alpha & \alpha & 1 & -2 & 0 & 1 \end{array}\right]$$

$\cdot Si\alpha \neq 0 \Rightarrow ec.4 : x_4 = 1$

$ec.3 : 9x_4 = -1 \Rightarrow$ no hay solución.

$\cdot$ Si$\alpha = 0 \Rightarrow$ infinitas soluciones

caso $\beta \neq 0$:

si $\alpha = 0 \Rightarrow ec.4 : 0\ x_4 = \beta$ con $\beta \neq 0 \Rightarrow$ no hay soluciones.
$\alpha \neq 0 \Rightarrow$ escalonamiento perfecto $\Rightarrow$ solución única.

caso $\alpha = 1, \beta = 1$:

$$\begin{bmatrix} -1 & 2 & 0 & 1 & | & 1 & | & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 & | & +1 & | & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 & | & 0 & | & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & | & 2 & | & 1 & -2 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 0 & 0 & | & 1 & | & 0 & -2 & 0 & 1 \\ 0 & 1 & -2 & 0 & | & +1 & | & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & | & -4 & | & -1 & 4 & 1 & -2 \\ 0 & 0 & 0 & 1 & | & 2 & | & 1 & -2 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 0 & 0 & | & 1 & | & 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & 0 & | & 9 & | & 1 & -7 & -2 & 4 \\ 0 & 0 & -1 & 0 & | & -4 & | & -1 & 4 & 1 & -2 \\ 0 & 0 & 0 & 1 & | & 2 & | & 1 & -2 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & | & 19 & | & 2 & -16 & -4 & 9 \\ 0 & 1 & 0 & 0 & | & 9 & | & 1 & -7 & -2 & 4 \\ 0 & 0 & -1 & 0 & | & -4 & | & -1 & 4 & 1 & -2 \\ 0 & 0 & 0 & 1 & | & 2 & | & 1 & -2 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 2 & -16 & -4 & 9 \\ 1 & -7 & -2 & 4 \\ 1 & -4 & -1 & 2 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

Solución sistema

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 19 \\ 9 \\ 4 \\ 2 \end{pmatrix}$$

Pregunta 2.

a)

$$\begin{aligned} p(x) &= q(x)(x^2-9)\cdot x + (2x^2-3) \\ p(x) &= \overline{q}(x)(x^2-9) + r(x) \end{aligned}$$

$$\Rightarrow \begin{array}{ccc} p(3) & = 2\cdot 9 - 3 & = 15 \\ p(-3) & = 2\cdot 9 - 3 & = 15 \end{array}$$

En segunda ecuación $gr\ (r(x)) \leq 1 \Rightarrow r(x) = a\cdot x + b$. Evaluando en $3y$ -3 la segunda ecuación

$$\begin{array}{ccccc} p(3) & = 15 & = & r(3) & = & 3a+b \\ p(-3) & = 15 & = & r(-3) & = & -3a+b \end{array}$$

$$\begin{aligned} \Rightarrow b &= 15 \\ a &= 0 \end{aligned}$$

$\Rightarrow r(x) = 15$

(hay al menos otra forma de hacerlo)

b)

(i) Como $p(x) \in I\!R[x] \Rightarrow$ si i es raíz $-i$ también y $1, 2, 3$.

$\Rightarrow (x-1)(x-2)(x-3)(x+i)(x-i)/$divide a $p(x)$

$\Rightarrow gr(p(x)) \geq 5$

(ii) $g(x) = (x-1)(x-2)(x-3)(x^2+1)$

Pregunta 3.

(i)

$$\begin{aligned}T(A+B) &= \sum_{i=1}^{n}(A+B)_{i,i} \\ &= \sum_{i=1}^{n}(A_{ii}+B_{ii}) \\ &= \sum_{i=1}^{n}A_{ii}+\sum_{i=1}^{n}B_{ii} = T(A)+T(B)\end{aligned}$$

(ii)

$$T(\lambda A) = \sum_{i=1}^{n}(\lambda A)_{ii} \qquad = \sum_{i=1}^{n}\lambda A_{i,i}$$
$$= \lambda\sum_{i=1}^{n}A_{ii} = \lambda T(A)$$

(iii)

$$\begin{aligned}T(A\cdot B) &= \sum_{i=1}^{n}(A\cdot B)_{i,i} \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}A_{ij}B_{ji} \\ &= \sum_{j=1}^{n}\sum_{i=1}^{n}B_{ji}\cdot A_{ij} \\ &= \sum_{j=1}^{n}(B\cdot A)_{jj} = T(B\cdot A)\end{aligned}$$

(iv) $T(P\cdot A\cdot P^{-1}) = T(P^{-1}\cdot P\cdot A) = T(I\cdot A) = T(A)$

(v)

$$\begin{aligned}T(A\cdot A^T) = \sum_{i=1}^{n}(A\cdot A^T)_{ii} &= \sum_{i=1}^{n}\sum_{j=1}^{n}A_{ij}A^T_{ji} \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}A^2_{ij}\end{aligned}$$

Como todo término es $\geq 0 \Rightarrow T(A \cdot A^T) \geq 0$

Si $A = 0 \Rightarrow A \cdot A^T = 0 \cdot 0 = 0 \Rightarrow T(0) = \sum_{i=1}^{n} 0 = 0$

$$\begin{aligned} T(A \cdot A^T) = 0 &\Leftrightarrow A_{ij}^2 = 0 \ \forall i, j \in \{1, \cdots, n\} \\ &\Leftrightarrow A = 0 \end{aligned}$$